

## dec revision 1 solutions

- [SQA] 1. (a) Express  $f(x) = x^2 - 4x + 5$  in the form  $f(x) = (x - a)^2 + b$ . 2
- (b) On the same diagram sketch:
- (i) the graph of  $y = f(x)$ ;
- (ii) the graph of  $y = 10 - f(x)$ . 4
- (c) Find the range of values of  $x$  for which  $10 - f(x)$  is positive. 1

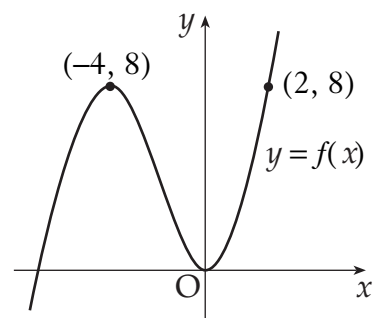
Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	2	C	NC	A5	$a = 2, b = 1$	2002 P1 Q7
(b)	4	C	NC	A3	sketch	
(c)	1	C	NC	A16, A6	$-1 < x < 5$	

<ul style="list-style-type: none"> <li>•<sup>1</sup> pd: process, e.g. completing the square</li> <li>•<sup>2</sup> pd: process, e.g. completing the square</li> <li>•<sup>3</sup> ic: interpret minimum</li> <li>•<sup>4</sup> ic: interpret <math>y</math>-intercept</li> <li>•<sup>5</sup> ss: reflect in <math>x</math>-axis</li> <li>•<sup>6</sup> ss: translate parallel to <math>y</math>-axis</li> <li>•<sup>7</sup> ic: interpret graph</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>a = 2</math></li> <li>•<sup>2</sup> <math>b = 1</math></li> <li>•<sup>3</sup> any two from: parabola; min. t.p. (2, 1); (0, 5)</li> <li>•<sup>4</sup> the remaining one from above list</li> <li>•<sup>5</sup> reflecting in <math>x</math>-axis</li> <li>•<sup>6</sup> translating +10 units, parallel to <math>y</math>-axis</li> <li>•<sup>7</sup> (-1, 5) i.e. <math>-1 &lt; x &lt; 5</math></li> </ul>
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[SQA] 2. The diagram shows a sketch of the function  $y = f(x)$ .

(a) Copy the diagram and on it sketch the graph of  $y = f(2x)$ .

(b) On a separate diagram sketch the graph of  $y = 1 - f(2x)$ .



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Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	2	B	CN	A3	sketch	2009 P1 Q23
(b)	3	B	CN	A3	sketch	

<ul style="list-style-type: none"> <li>•<sup>1</sup> ic: scaling parallel to x-axis</li> <li>•<sup>2</sup> ic: annotate graph</li> <li>•<sup>3</sup> ss: correct order for refl(x) and trans</li> <li>•<sup>4</sup> ic: start to annotate final sketch</li> <li>•<sup>5</sup> ic: complete annotation</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> sketch and one of (0,0), (1,8), (-2,8)</li> <li>•<sup>2</sup> remaining points</li> <li>•<sup>3</sup> reflect in x-axis then vertical translation</li> <li>•<sup>4</sup> sketch and one of (0,1), (1,-7), (-2,-7)</li> <li>•<sup>5</sup> remaining points</li> </ul>
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[SQA] 3. Differentiate  $\sin 2x + \frac{2}{\sqrt{x}}$  with respect to  $x$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C3		1989 P1 Q10
	2	A/B	NC	C20		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2x^{-\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>\cos 2x</math></li> <li>•<sup>3</sup> <math>\times 2</math></li> <li>•<sup>4</sup> <math>-x^{-\frac{3}{2}}</math></li> </ul>
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- [SQA] 4. Find the coordinates of the point on the curve  $y = 2x^2 - 7x + 10$  where the tangent to the curve makes an angle of  $45^\circ$  with the positive direction of the  $x$ -axis.

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Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	G2, C4	(2, 4)	2002 P1 Q4
<ul style="list-style-type: none"> <li>•<sup>1</sup> sp: know to diff., and differentiate</li> <li>•<sup>2</sup> pd: process gradient from angle</li> <li>•<sup>3</sup> ss: equate equivalent expressions</li> <li>•<sup>4</sup> pd: solve and complete</li> </ul>				<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = 4x - 7</math></li> <li>•<sup>2</sup> <math>m_{\text{tang}} = \tan 45^\circ = 1</math></li> <li>•<sup>3</sup> <math>4x - 7 = 1</math></li> <li>•<sup>4</sup> (2, 4)</li> </ul>		

- [SQA] 5. Find the  $x$ -coordinate of each of the points on the curve  $y = 2x^3 - 3x^2 - 12x + 20$  at which the tangent is parallel to the  $x$ -axis.

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Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4		1993 P1 Q4
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>6x^2 - 6x - 12</math></li> <li>•<sup>3</sup> <math>\dots\dots = 0</math></li> <li>•<sup>4</sup> <math>x = -1, 2</math></li> </ul>						

- [SQA] 6. (a)  $f(x) = 2x + 1$ ,  $g(x) = x^2 + k$ , where  $k$  is a constant.
- (i) Find  $g(f(x))$ . (2)
  - (ii) Find  $f(g(x))$ . (2)
- (b) (i) Show that the equation  $g(f(x)) - f(g(x)) = 0$  simplifies to  $2x^2 + 4x - k = 0$ . (2)
- (ii) Determine the nature of the roots of this equation when  $k = 6$ . (2)
  - (iii) Find the value of  $k$  for which  $2x^2 + 4x - k = 0$  has equal roots. (3)

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(a)	4	C	NC	A4		1996 P2 Q4
(b)	7	C	NC	A17, A18, A6		

<p>(a)</p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>g(2x+1)</math></li> <li>•<sup>2</sup> <math>(2x+1)^2 + k</math></li> <li>•<sup>3</sup> <math>f(x^2+k)</math></li> <li>•<sup>4</sup> <math>2(x^2+k)+1</math></li> </ul>	<p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>5</sup> <math>4x^2 + 4x + k + 1</math> AND <math>2x^2 + 2k + 1</math></li> <li>•<sup>6</sup> <math>4x^2 + 4x + k + 1 - (2x^2 + 2k + 1) = 0</math> so <math>2x^2 + 4x - k = 0</math></li> <li>•<sup>7</sup> <math>b^2 - 4ac = 16 - 4 \times 2 \times (-6) = 64</math></li> <li>•<sup>8</sup> so roots real &amp; distinct</li>   <li>•<sup>9</sup> <math>b^2 - 4ac = 16 - 4 \times 2 \times (-k)</math></li> <li>•<sup>10</sup> <math>b^2 - 4ac = 0</math> for equal roots</li> <li>•<sup>11</sup> <math>k = -2</math></li> </ul>
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- [SQA] 7. Show that the equation  $(1 - 2k)x^2 - 5kx - 2k = 0$  has real roots for all integer values of  $k$ .

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Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	A/B	CN	A18, A16, CGD	proof	2002 P2 Q9

<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: know to use discriminant</li> <li>•<sup>2</sup> ic: pick out discriminant</li> <li>•<sup>3</sup> pd: simplify to quadratic</li> <li>•<sup>4</sup> ss: choose to draw table or graph</li> <li>•<sup>5</sup> pd: complete proof using <math>\text{disc.} \geq 0</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> discriminant = ...</li> <li>•<sup>2</sup> <math>\text{disc} = (-5k)^2 - 4(1 - 2k)(-2k)</math></li> <li>•<sup>3</sup> <math>9k^2 + 8k</math></li> <li>•<sup>4</sup> e.g. draw a table, graph, complete the square</li> <li>•<sup>5</sup> complete proof and conclusion relating to <math>\text{disc.} \geq 0</math></li> </ul>
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- [SQA] 8. When  $f(x) = 2x^4 - x^3 + px^2 + qx + 12$  is divided by  $(x - 2)$ , the remainder is 114. One factor of  $f(x)$  is  $(x + 1)$ . Find the values of  $p$  and  $q$ .

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Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	C	CN	A21		1991 P1 Q6
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(2) = 114</math></li> <li>•<sup>2</sup> <math>f(-1) = 0</math></li> <li>•<sup>3</sup> <math>4p + 2q = 78</math></li> <li>•<sup>4</sup> <math>p - q = -15</math></li> <li>•<sup>5</sup> <math>p = 8, q = 23</math></li> </ul>						

- [SQA] 9. Given that  $\tan \alpha = \frac{\sqrt{11}}{3}$ ,  $0 < \alpha < \frac{\pi}{2}$ , find the exact value of  $\sin 2\alpha$ .

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Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	NC	T8		1995 P1 Q12
<ul style="list-style-type: none"> <li>•<sup>1</sup> "third side" = <math>\sqrt{20}</math></li> <li>•<sup>2</sup> <math>\sin \alpha = \frac{\sqrt{11}}{\sqrt{20}}</math> or <math>\cos \alpha = \frac{3}{\sqrt{20}}</math></li> <li>•<sup>3</sup> <math>2 \times \frac{\sqrt{11}}{\sqrt{20}} \times \frac{3}{\sqrt{20}}</math></li> </ul>						

- [SQA] 10.

(a) Show that  $2 \cos 2x^\circ - \cos^2 x^\circ = 1 - 3 \sin^2 x^\circ$ .

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(b) Hence solve the equation  $2 \cos 2x^\circ - \cos^2 x^\circ = 2 \sin x^\circ$  in the interval  $0 \leq x < 360$ .

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Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	1	C	CR	T8		1997 P1 Q18
(a)	1	A/B	CR	T8		
(b)	1	C	CR	T10		
(b)	3	A/B	CR	T10		
<ul style="list-style-type: none"> <li>•<sup>1</sup> substitute <math>1 - 2 \sin^2 x^\circ</math> for <math>\cos 2x^\circ</math></li> <li>•<sup>2</sup> substitute <math>1 - \sin^2 x^\circ</math> for <math>\cos^2 x^\circ</math></li> <li>•<sup>3</sup> <math>3 \sin^2 x^\circ + 2 \sin x^\circ - 1 = 0</math></li> <li>•<sup>4</sup> <math>(3 \sin x^\circ - 1)(\sin x^\circ + 1) = 0</math></li> <li>•<sup>5</sup> <math>\sin x^\circ = \frac{1}{3}, -1</math></li> <li>•<sup>6</sup> <math>19.5, 160.5, 270</math></li> </ul>						

11. Solve  $2 \cos 2x - 5 \cos x - 4 = 0$  for  $0 \leq x < 2\pi$ .

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Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	B	CN	T10, T7	$x = 2.419, 3.864$	2010 P2 Q4
<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: know to use double angle formula</li> <li>•<sup>2</sup> ic: express as quadratic in <math>\cos x</math></li> <li>•<sup>3</sup> ss: start to solve</li> <li>•<sup>4</sup> pd: reduce to equations in <math>\cos x</math> only</li> <li>•<sup>5</sup> pd: complete solutions to include only one where <math>\cos x = k</math> with <math> k  &gt; 1</math></li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2 \times (2 \cos^2 x - 1) \dots</math></li> <li>•<sup>2</sup> <math>4 \cos^2 x - 5 \cos x - 6 = 0</math></li> <li>•<sup>3</sup> <math>(4 \cos x + 3)(\cos x - 2) = 0</math></li> <li>•<sup>4</sup> <math>\cos x = -\frac{3}{4}</math> and <math>\cos x = 2</math></li> <li>•<sup>5</sup> 2.419, 3.864 and no solution.</li> </ul>						

[SQA] 12. A is the point  $(2, -5, 6)$ , B is  $(6, -3, 4)$  and C is  $(12, 0, 1)$ . Show that A, B and C are collinear and determine the ratio in which B divides AC.

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Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	4	C	CN	G23, G25	2 : 3 or equivalent	1996 P1 Q6
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\vec{AB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}</math> or <math>\vec{AC} = \begin{pmatrix} 10 \\ 5 \\ -5 \end{pmatrix}</math> or <math>\vec{BC} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}</math></li> <li>•<sup>2</sup> <math>\vec{AB} = 2 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}</math> and <math>\vec{BC} = 3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}</math> or equivalent</li> <li>•<sup>3</sup> <math>AB \parallel BC</math> and B is point in common</li> <li>•<sup>4</sup> 2:3 (or equivalent e.g. <math>1:1\frac{1}{2}</math>)</li> </ul>						

[END OF QUESTIONS]